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Interaction of Sonic Transverse Jets with Supersonic External Flows

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This paper presents a theoretical analysis for the determination of the two-dimensional flowfield in the vicinity of the injection port of a secondary sonic jet normal to a supersonic external stream. The solution is valid for flows in which the boundary layer is nonexistent and mixing effects can be neglected. The gas is assumed to be perfect and inviscid. The method of characteristics was employed in the determination of the jet flowfield, and the time-dependent finite-difference method was employed to determine the external flowfield. The key to the over-all solution lies in coupling the flowfields through the jet boundary. This is accomplished by constraining the static pressure to be continuous across the jet boundary. The coupling technique developed herein has proven successful, and the entire theory, through the use of the coupling technique, has been formulated into a computer program. Extensive calculations have been carried out to study the effect of freestream Mach number, jet total pressure and ratio of specific heats on penetration, shock stand-off distance, and upstream pressure distribution.

Introduction

WHEN a secondary jet issues laterally into a supersonic stream, the interaction between the two streams creates a high pressure region on the wall surface in front of the jet port. The resulting force due to this high pressure can be several times larger than the thrust of the jet alone. Therefore, the interaction mechanism presents itself as a very useful technique for the vector control of high-speed flight vehicles. In addition, the secondary jet device has several other advantages over the conventionally used fins, such as 1) eliminating the aerodynamic heating problem on the control surfaces, 2) being effective at all altitudes, and 3) possibility of a weight saving design. In order to employ the jet control properly, prediction of shock behavior, static pressure distribution on the surface, and all the flow properties in the neighborhood of the jet exit is necessary. In the past few years, an intensive effort has been devoted to this problem by many research groups. However, this effort has been primarily experimental in nature with the goal of obtaining fundamental information, similarity rule,[1–8] drawing analogies,[9–12] developing simple analytic models[13–17] (discussion of previous work as presented in Ref. 18). To date, no analytical method has been published that presents a direct approach for the determination of shock behavior or other flow properties. The purpose of this paper is to present the result of a theoretical approach to this problem. This approach is based on the inviscid flow assumption, i.e., nonexistence of boundary layer. The flow model is depicted in Fig. 1. The solution is developed by coupling the jet flow region, in which the method of characteristics is applicable, to the external flow region determined by the application of the time-dependent finite-difference method. This flow model may seem unrealis-

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tic, since the boundary layer does exist in the real flow and the boundary-layer separation can quite often be detected. Consequently, the flowfield upstream of the jet will be similar to a flow over a forward facing step rather than the one described herein, and this approach will, as expected, overestimate the contribution of upstream over-pressure to the jet side force. However, the present approach can be viewed as a first step toward solving the over-all problem and is primarily intended to provide an understanding of the interaction between the jet and external flow through the inviscid flow solution.

This study was motivated by the results of an experimental study by Amick and Hays.[19] In their study, they conclude that the flow structure near the nozzle exit is similar to the flow pattern which results when the jet is replaced by a solid cylinder perpendicular to the freestream. This implies that the mixing between the injectant and the external stream is negligible in the immediate neighborhood of the injector. This finding was confirmed later by Orth[10] for the range $[(X/d_j) < 10]$. If the mixing effect is small enough to be neglected, the whole flowfield can be divided into two parts (i.e., the jet flowfield and the external flowfield) and solved accordingly. However, a problem arises, because the two flowfields have one common boundary yet unknown. This boundary is defined by the condition that the static pressure should be equal in the flowfields on both sides of the boundary. Obviously, the problem must be approached by iteration techniques.

The additional assumptions employed in this study are that the gas is ideal and the flow is two-dimensional. The equations for the coupled method of characteristics solution region and the time-dependent finite difference method region have been programed for the IBM 360/50 digital computer. Extensive calculations have been carried out to study the effect of freestream Mach number, jet total pressure and injectant composition on penetration, shock stand-off distance, and upstream pressure distribution.

Theoretical Analysis

The key element in the analysis to be presented lies in the coupling of the jet flowfield to the external flowfield across the interaction shock. The jet flowfield and the primary (or external) flowfield will be discussed separately; then, these will be coupled to obtain an over-all description of the general flowfield that results when a sonic jet issues into a supersonic flow.

Jet Flowfield

The method of characteristics along with the Rankine-Hugoniot equations have been employed in the jet flowfield analysis. The computation is initiated from the nozzle exit plane; the flow is assumed to be uniform and at a slightly

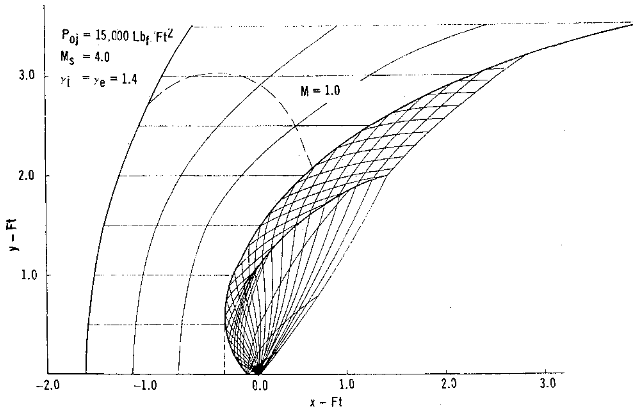

Fig. 1 Schematic representation of the proposed flow model.

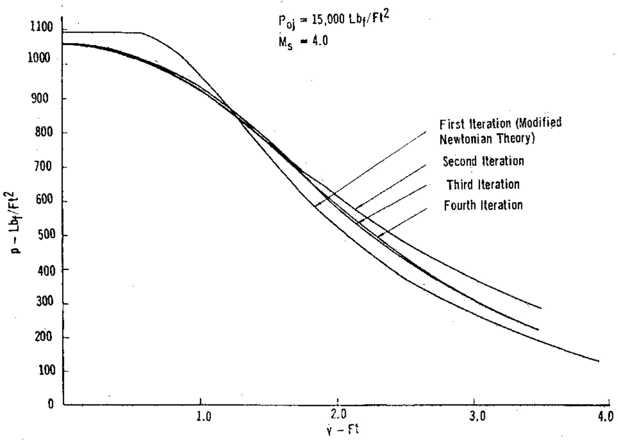

Fig. 2 Jet boundary pressure distribution for various iterations.

supersonic condition ($M = 1.02$ is employed in the work presented here). The flowfield points are computed along left-running characteristics. When characteristics of the same family intersect, it is an indication of the presence of a shock. The flow properties across the shock can be computed by using the Rankine-Hugoniot equations. A numerical scheme developed by Eastman[20] has been adopted in the shock point calculations.

When a left-running characteristic intersects the jet boundary, the static pressure inside the jet boundary is iterated until it matches the external static pressure, since the static pressure should be continuous across the boundary.

External Flowfield

The assumption of no mixing between the primary flow and the secondary jet permits the jet to be treated as a blunt body immersed in the primary flow, with the blunt body shape coinciding with the shape of the jet as determined in the preceding section. The primary flow can then be determined by using blunt body theories.

The determination of fluid properties in the subsonic and transonic regions over a blunt body has been the focus of numerous investigations in gasdynamics. Methods in current use can be classified into two categories: inverse methods and direct methods. Although the inverse solution is numerically exact, the extreme sensitivity of the resulting body shape with respect to the assumed shock shape makes the indirect method less appealing for the present investigation. However, in the past, the direct method also encountered a grave difficulty, in terms of computer storage and computation time, because a great number of points was required to describe the flowfield (the shock thickness must be of the order of the net spacing). Recently, Moretti and Abbett[21,22] have effectively overcome this difficulty through the development of a technique which permits the shock to be considered as a boundary condition. This treatment markedly reduces the number of calculation points and, in particular, eliminates the need for calculations upstream of the shock. This method has been adopted in the present study.

The governing equations used in this method are unsteady, two-dimensional continuity, momentum and energy equations. The important feature of this method lies in imposing the condition that the tangential velocity components be the same on both sides of the shock. If a negative value of the tangential velocity component is superimposed on the flowfield, the flowfield in the neighborhood of the shock point becomes quasi-one-dimensional, and a general, one-dimensional, unsteady shock treatment can be applied in the neighborhood of the shock point. A detailed explanation of this concept and the basic equations are presented in Refs. 21 and 22.

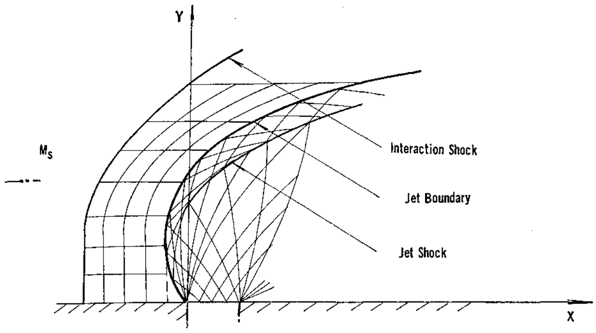

Fig. 3 Flow pattern of a typical jet interaction flowfield.

Coupling Technique

The critical step in the technique developed herein lies in the eventual coupling of the solutions for the jet flowfield and the external flowfield previously described. The nature of coupling demands that the two flowfields be subjected to a common, but unknown, boundary condition; i.e., that no static pressure discontinuity exists at the jet boundary. Two methods for accomplishing this have been developed during the course of this investigation. These two techniques will be presented in the following paragraphs.

In order to establish an initial static pressure distribution on the jet boundary, the modified Newtonian theory can be employed in the jet boundary calculations. As soon as the method of characteristic solution provides a jet configuration, the time-dependent finite-difference method can be used to determine the external flowfield, including the pressure distribution along the jet boundary. This pressure distribution would then replace the modified Newtonian solution as a second guess for the jet flow calculations. This iterative process can be repeated cyclically until the pressure difference between two consecutive iterations becomes less than 5% of the boundary pressure. This technique has been proven to be successful for general cases, but it causes some fluctuation in pressure distribution in a few cases where the freestream Mach number is higher than 3. For this reason, a different approach was developed.

The second technique developed, which is actually a modification of the first technique, effectively eliminates the problem of strong fluctuations in pressure distribution during the iteration process for coupling the flowfields. This approach follows the same procedure described for the first method until the second iteration is completed. Then, instead of directly taking the pressure distribution obtained from the external flowfield, the average value of two consecutive iterations is used for a new jet boundary-pressure distribution. This technique serves as a damping device in the iteration

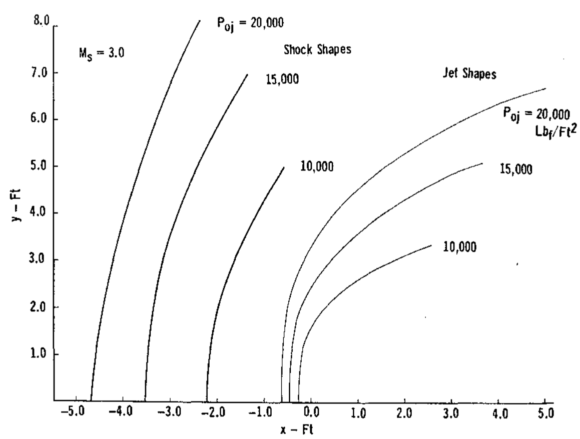

Fig. 4 Variations of shock shapes corresponding to various jet shapes (Ms = 3.0).

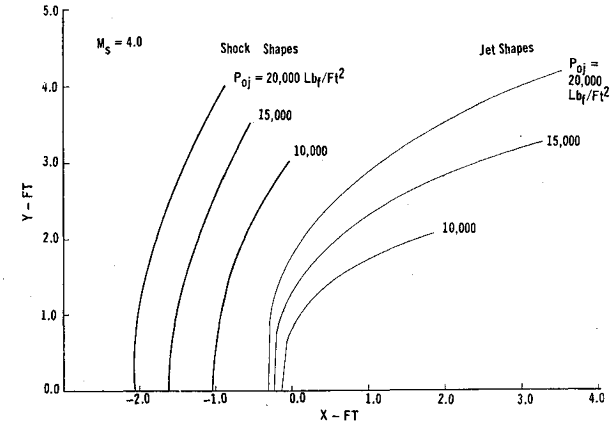

Fig. 5 Variations of shock shapes corresponding to various jet shapes.

process and has proved to be very successful. A typical pressure distribution behavior after 4 iterations is illustrated in Fig. 2.

The final solution presented herein is completely independent of the modified Newtonian theory. It is worth noting, however, that the modified Newtonian theory does give a reasonably good approximation, even in the type of flow investigated herein (see Fig. 2).

A computed jet and shock shape is illustrated in Fig. 3. The characteristic and the external mesh constructions are also shown in this figure. It should be mentioned that the jet boundary near the nozzle exit has been slightly modified to eliminate the concave tip. Bastianon[23] indicated that a periodic behavior may occur on a concave blunt body; however, no experimental evidence of such behavior in a jet interaction flowfield has been reported. Therefore, it is assumed that such motion does not play a major role in the flow system under study. The method of modification is simply to replace the small segment by a parabola which is tangent to the jet boundary and normal to the solid surface. This small segment is shown as a dashed line in Fig. 3.

Discussion of Results

All of the equations as well as the numerical schemes used in this study have been programed for the IBM 360/50 digital computer. For illustrative purposes, sample cases have been calculated to study the effects of the freestream Mach number M_S, the jet total pressure P_{oj}, and the ratio of specific heats on a two-dimensional flowfield. For the sake of comparison, certain variables were held constant in the results presented in this section. These are as follows: slot nozzle width = 0.1 ft; freestream specific heat ratio = 1.4; freestream static pressure = 50.7 lbf/ft²; and freestream density = 0.00684 lbm/ft³.

Theoretical predictions of the jet boundaries for three different values of jet total pressure are shown in Figs. 4 and 5 at fixed M_S (M_S = 3,4). It is clearly shown that the secondary jet penetrates higher into the primary flowfield as the P_{oj} jet increases. Also, the interaction shock shape and the stand-off distance are very sensitive to the jet shape and the M_S. When the P_{oj} increases, the shock is pushed up-

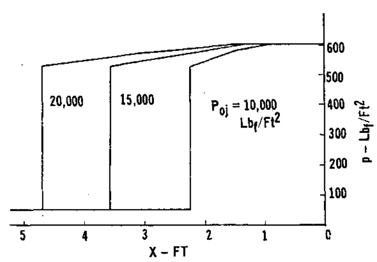

Fig. 6 Pressure distribution upstream of a jet for various jet total pressures (Ms = 3.0).

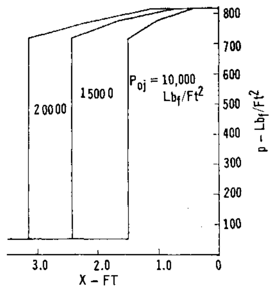

Fig. 7 Pressure distribution upstream of a jet for various jet total pressures (Ms = 3.5).

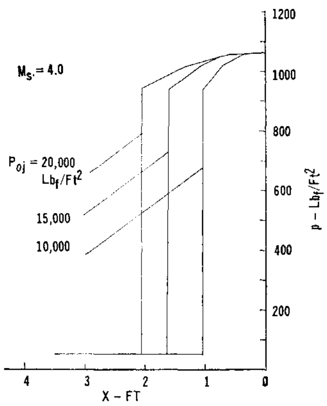

Fig. 8 Pressure distribution upstream of a jet for various jet total pressures (Ms = 4.0).

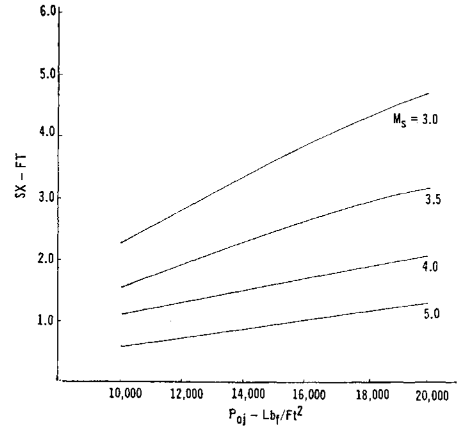

Fig. 10 Variations of interaction shock stand-off distance for various jet total pressures under various freestream Mach number conditions—Mach number as the parameter.

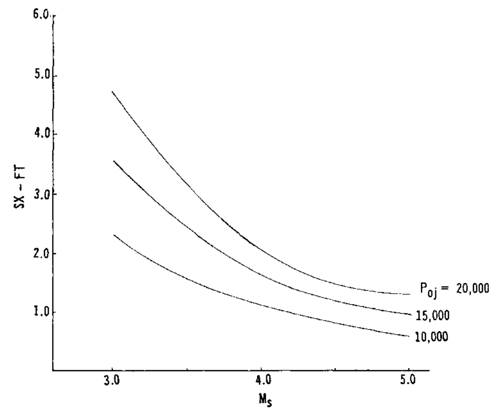

Fig. 11 Variations of interaction shock stand-off distance for various jet total pressure under various freestream Mach number conditions—Mach number as the parameter.

stream; and, in fact, when the total pressure is doubled, the stand-off distance is approximately doubled.

The interaction shock generates a high pressure region ahead of the jet, and this high pressure region is one of the major contributions to lateral thrust. For a fixed M_S, static pressure distributions obtained from the present solution are shown in Figs. 6–9.

Another important observation is illustrated in Fig. 10. Here, the rise of the shock stand-off distance SX with the P_{oj} (taking the M_S as a parameter) is clearly demonstrated. It may be noted that the stand-off distance increases almost linearly with the jet total pressure. Figure 11 is a cross-plot of Fig. 10, which elucidates the effect of the M_S on the stand-off distance.

In order to study the effect of the injectant gas composition, calculations were performed with the ratio of the specific heats (γ_j) of the injectant gas as the parameter. The result of these calculations is summarized in Fig. 12. A slight increase in the SX (i.e., extent of the high pressure region) for the injectant gas with the lower value of γ_j is evident.

As mentioned previously, the solution presented herein is an inviscid solution which does not take into account the upstream boundary-layer separation and the mixing effect. Currently, no theoretical method has been reported for rigorously determining the effect of separation and mixing on the upstream flowfield. Obviously, an exact correlation with experimental results is difficult, if not impossible, because the upstream conditions (which are the input conditions for the present solution) remain unknown. It is the authors' intention to delay reporting the experimental correlation until further studies employing an extension of the current inviscid solution are completed.

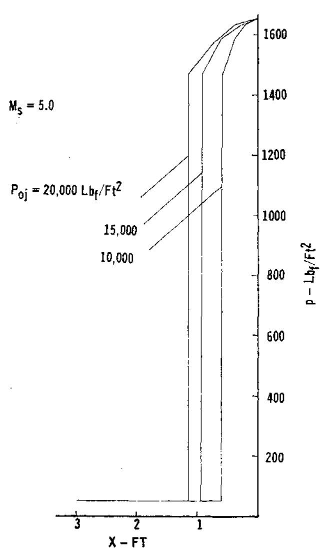

Fig. 9 Pressure distributions upstream of a jet for various jet total pressures (Ms = 5.0).

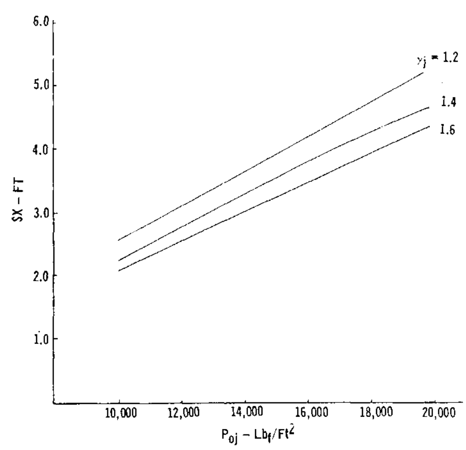

Fig. 12 Variations of interaction shock stand-off distance for various jet ratio of specific heats.

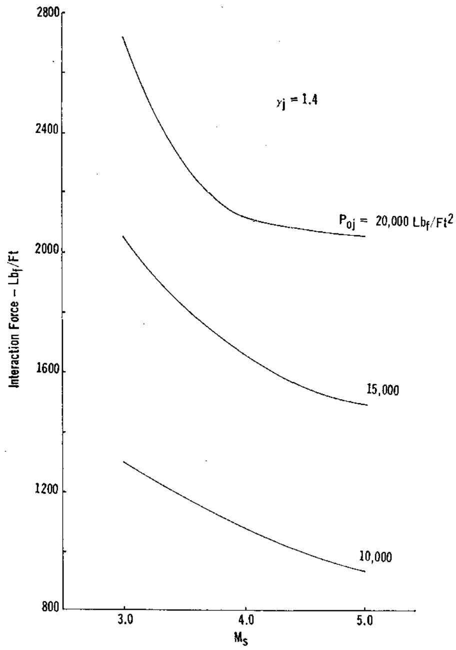

Fig. 13 Variation of interaction force for various Mach number and jet total pressure.

Finally, the interaction force is obtained by integrating the high pressure between the interaction shock and the jet port. The effects of Mach number and P_{oj} on this force are shown in Fig. 13.

Conclusions

The method presented herein provides a numerical solution for the determination of the two-dimensional flowfield around the injection port of a secondary sonic jet normal to a supersonic external stream. The method of characteristics was used to analyze the jet flowfield, and the time-dependent finite-difference method was employed to determine the external flowfield. The key to the over-all solution lay in developing a procedure for coupling the jet flowfield with the external flowfield. The coupling technique developed has proved to be successful, and the theory has been formulated into a computer program. An extensive calculation has been conducted and the results of the calculations indicate the following:

1) The secondary jet penetrates higher into the primary flowfield as the P_{oj} increases and the M_S decreases.

2) The interaction SX increases when the P_{oj} increases. In fact, the stand-off distance almost increases linearly with the P_{oj}.

3) The extent of the upstream high-pressure region increases as the P_{oj} increases, but the extent of the high-pressure region decreases and the magnitude of the pressure increases as the M_S increases.

4) The interaction SX increases as the ratio of specific heats of the injectant gas decreases.

5) The interaction force increases as the P_{oj} increases, but decreases as the M_S increases.

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